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## Application of the $k$ - $\omega$ Turbulence Model to Quasi-Three-Dimensional Turbomachinery Flows

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### Introduction

**M**ANY computational fluid dynamics codes for turbomachinery use the Baldwin-Lomax (B-L) turbulence model.<sup>1</sup> It is easy to implement in two dimensions and works well for predicting overall turbomachinery performance. However, it is awkward to implement in three dimensions, often has difficulty finding the length scale, has a crude transition model, and neglects freestream turbulence, surface roughness, and mass injection.

The  $k$ - $\omega$  model developed by Wilcox<sup>2</sup> is an appealing alternative for several reasons. First, it is the only two-equation model that can be integrated to the wall without requiring damping functions or the distance to the wall, and hence,

should behave well numerically. Second, the effects of free-stream turbulence, surface roughness, and mass injection are easily included in the model. Finally, transition can be simulated using the low Reynolds number version of the model.<sup>3</sup>

Menter applied the  $k$ - $\omega$  model to external flows and showed very good results for flows with adverse pressure gradients.<sup>4</sup> Liu and Zheng<sup>5</sup> described their implementation of the  $k$ - $\omega$  model in a cascade code that included an area change term to account for endwall convergence. They validated the model for a flat plate, and compared the B-L and  $k$ - $\omega$  models to measured surface pressures for a low-pressure turbine cascade. Since they did not use the low Reynolds number version of the model, their results showed problems resulting from early transition.

In this Note the low Reynolds number  $k$ - $\omega$  model was incorporated in the author's quasi-three-dimensional turbomachinery analysis code.<sup>6</sup> The code includes the effects of rotation, radius change, and stream-surface thickness variation, and also includes the B-L turbulence model. The  $k$ - $\omega$  model was implemented using many of Menter's<sup>4</sup> recommendations and an implicit approximate-factorization scheme described by Baldwin and Barth.<sup>7</sup> The model was tested for a transonic compressor with rotation and variable stream-surface radius and height, and for a transonic turbine vane with transition and heat transfer. Results were compared to the B-L model and to experimental data.

### Quasi-Three-Dimensional Navier-Stokes Code

The quasi-three-dimensional Navier-Stokes equations have been developed for an axisymmetric stream surface in an  $(m, \theta)$  coordinate system as shown in Fig. 1. The meridional coordinate  $m$  is given by  $dm^2 = dz^2 + dr^2$ , and the tangential coordinate  $\theta$  rotates with the blade row with angular velocity  $\Theta$ . The radius  $r$  and the thickness  $h$  of the stream surface are assumed to be known functions of  $m$ . The equations were mapped to a body-fitted coordinate system, nondimensional-

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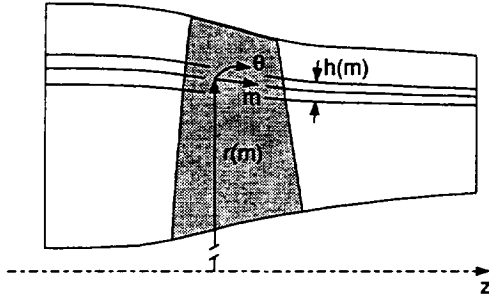


Fig. 1 Quasi-three-dimensional stream surface for a compressor rotor.

ized, and simplified using the thin-layer approximation. The flow equations and details of the explicit Runge-Kutta scheme are given in Ref. 6.

### $k$ - $\omega$ Turbulence Model

The  $k$ - $\omega$  turbulence model solves two transport equations for the turbulent kinetic energy  $k$  and the specific dissipation rate  $\omega$ . A basic formulation is for fully turbulent flows and a low Reynolds number formulation is used for transition.<sup>3</sup> The quasi-three-dimensional form of the low Reynolds number  $k$ - $\omega$  equations is as follows:

$$\partial_t q + U \partial_\xi q + V \partial_\eta q - \frac{J}{\rho Re} G = \frac{1}{\rho} (P - D) \quad (1)$$

$$q = [k, \omega]^T \quad (2)$$

$$\mu_\tau = \alpha^* (\rho k / \omega) \quad (3)$$

The  $r$  and  $h$  appear in the  $\theta$  metrics and the Jacobian, and  $\Theta$  appears in the relative contravariant velocities  $U$  and  $V$ :

$$\begin{bmatrix} \xi_m & \xi_\theta \\ \eta_m & \eta_\theta \end{bmatrix} = J \begin{bmatrix} \theta_\eta & -m_\eta/r \\ -\theta_\xi & m_\xi/r \end{bmatrix} \quad (4)$$

$$J = [rh(m_\xi \theta_\eta - m_\eta \theta_\xi)]^{-1}$$

$$U = \xi_m u + \xi_\theta (\nu - r\Theta) \quad (5)$$

$$V = \eta_m u + \eta_\theta (\nu - r\Theta)$$

The diffusive terms  $G$  use the thin-layer approximation:

$$G = \frac{(\eta_m^2 + \eta_\theta^2)}{J} \begin{bmatrix} (\mu + \sigma^* \mu_\tau) \partial_\eta k \\ (\mu + \sigma \mu_\tau) \partial_\eta \omega \end{bmatrix} \quad (6)$$

The production terms  $P$  use the vorticity  $\Omega$  as suggested by Menter<sup>4</sup>:

$$\frac{P}{\rho} = \begin{bmatrix} \frac{\mu_\tau}{\rho Re} \Omega^2 - \frac{2}{3} k \left( \partial_m u + \frac{1}{r} \partial_\theta \nu \right) \\ \alpha \left[ \alpha^* \Omega^2 - \frac{2}{3} \omega \left( \partial_m u + \frac{1}{r} \partial_\theta \nu \right) \right] \end{bmatrix} \quad (7)$$

$$\Omega = \partial_m \nu - \frac{1}{r} \partial_\theta u + \frac{\nu}{r} d_m r$$

The destruction terms  $D$  are given by

$$\frac{D}{\rho} = \begin{bmatrix} \beta^* \omega k \\ \beta \omega^2 \end{bmatrix} \quad (8)$$

The baseline  $k$ - $\omega$  model has five coefficients,  $\beta = 3/40$ ,

$\beta^* = 9/100$ ,  $\sigma = 1/2$ ,  $\sigma^* = 1/2$ , and  $\alpha = 5/9$ , and the trivial constant  $\alpha^* = 1$ . The low Reynolds number model replaces three of the constants with  $\beta^* = (9/100)F_\beta$ ,  $\alpha = (5/9)(F_\alpha/F_\mu)$ , and  $\alpha^* = F_\mu$ , where  $F_i$  are the following bilinear functions of the turbulence Reynolds number  $Re_\tau$ :

$$F_\beta = \frac{5/18 + (Re_\tau/R_\beta)^4}{1 + (Re_\tau/R_\beta)^4}$$

$$F_\alpha = \frac{\alpha_0 + Re_\tau/R_w}{1 + Re_\tau/R_w}$$

$$F_\mu = \frac{\alpha_0^* + Re_\tau/R_k}{1 + Re_\tau/R_k}$$

$$Re_\tau = \frac{\rho k}{\mu_\tau \omega}$$

with  $\alpha_0 = 1/10$ ,  $\alpha_0^* = \beta/3 = 1/40$ , and  $R_\beta = 8$ ,  $R_w = 27/10$ ,  $R_k = 6$ .

### Boundary Conditions

At the inlet the turbulence intensity  $Tu$  and turbulent viscosity  $\mu_\tau$  were specified. Then  $\omega$  was found from Eq. (3) and  $k$  was found from

$$k = \frac{1}{2} Tu^2 U_{in}^2 \quad (10)$$

On solid walls  $k = 0$ , and  $\omega$  was set using Wilcox's roughness model:

$$\omega_{wall} = \frac{\mu_\tau}{\nu} \times \begin{cases} \left( \frac{50}{k_r^+} \right)^2 & k_r^+ < 25 \\ \frac{100}{k_r^+} & k_r^+ \geq 25 \end{cases} \quad (11)$$

The equivalent sand grain roughness height  $k_r^+$  was set to five, giving a hydraulically smooth surface.

Values of  $k$  and  $\omega$  were extrapolated at the exit and treated as periodic across trailing-edge wake cut lines and between blade rows.

### Approximate Factorization Solution Scheme

An implicit approximate-factorization scheme, described by Baldwin and Barth,<sup>7</sup> was used to solve the  $k$ - $\omega$  equations. Advective terms were approximated using first-order upwind differences and diffusive terms were approximated by second-order central differences. The destruction terms were linearized as suggested by Menter<sup>4</sup> and were included in the streamwise  $\xi$  factor. The production terms were treated explicitly. The  $k$  and  $\omega$  equations were solved uncoupled from each other and from the flow equations.

The CPU time for one iteration of the  $k$ - $\omega$  model was only 5% more than that for one application of the B-L model. Experience has shown that it is sufficient to update the B-L model every five iterations. In general, it was necessary to update the  $k$ - $\omega$  model every two iterations with twice the time step of the flow solution, making a  $k$ - $\omega$  solution about 18% slower than a B-L solution.

### Transonic Compressor Rotor Wake

The transonic compressor rotor studied by Suder et al.<sup>8</sup> was used to test the quasi-three-dimensional effects in the model. This rotor was used for a three-dimensional blind test case for turbomachinery codes organized by ASME and the International Gas Turbine Institute. Many of the codes used for the test case were unable to predict the measured wake spreading and decay. It was conjectured that the B-L model was re-

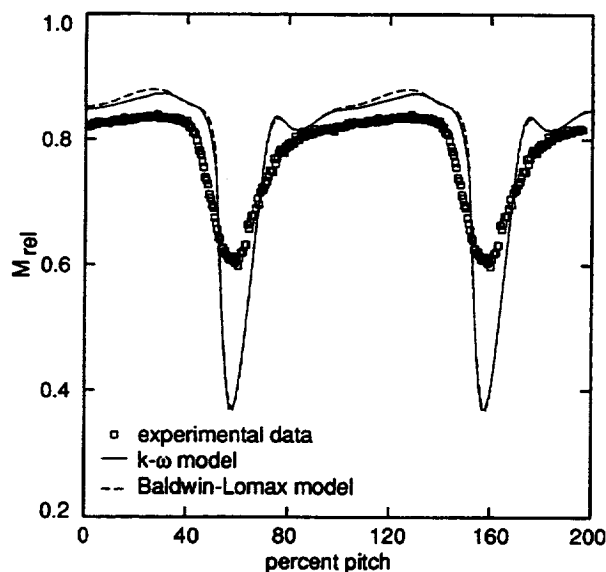


Fig. 2 Computed and measured near-wake Mach number profiles for a transonic compressor rotor.

sponsible for the poor predictions and hoped that the  $k-\omega$  model would improve the results.

A section of the rotor at 70% span was analyzed. The stream-surface  $r$  and  $h$  were specified to model the converging endwalls. The radius was specified as a line at 70% of local span, and  $h$  was specified as the local span normalized by the inlet span. A meridional view of the stream surface is shown in Fig. 1. The choice of  $r$  and  $h$  can have a first-order effect on the solution. However, the algebraic guess used here often gives reasonable results that can be used to compare turbulence models to each other.

A C-type grid with  $319 \times 45$  points and a wall spacing of  $y^+ < 2$  was used. The calculations were run 2000 iterations, which took about 3.25 min for the  $k-\omega$  model on a Cray C-90 computer.

Figure 2 compares computed and measured Mach number profiles 0.28 chords downstream of the trailing edge. The  $k-\omega$  (solid line) and B-L (dashed) results are nearly identical. The computed Mach number in the core flow is slightly high as a result of the specified stream surface, and the computed wakes are narrower and deeper than the measured wakes. Varying  $\omega_{in}$  by five orders of magnitude and varying  $Tu$  from 3 to 6% had little effect on the computed wake spreading. It is now thought that the measured wake spreading may be caused by unsteady vortex shedding from the trailing edge.

### Transonic Turbine Cascade

A transonic turbine vane tested by Arts et al.<sup>9</sup> was also computed. Comparisons were made with measured surface pressures and heat transfer data. A C-type grid with  $383 \times 49$  points and a wall spacing of  $y^+ < 1.5$  were used. Surface heat transfer converged to plotting accuracy in 5000 iterations, which took about 8 min on the Cray C-90.

Computed distributions of isentropic surface Mach number are compared to experimental data for exit Mach numbers of 0.875 and 1.02 in Fig. 3. The B-L and  $k-\omega$  models gave identical results. The subsonic results agree very well with the experimental data. The transonic results slightly underpredict the Mach number on the rear (uncovered) part of the suction surface.

Figure 4 compares measured and computed values of surface heat transfer coefficient  $H$  [ $W/(m^2 K)$ ] vs distance  $S$  (mm) along the vane surface for a case with exit  $M_{exit} = 0.875$ ,  $Re_{exit} = 1 \times 10^6$ , and  $Tu = 4\%$ . The experimental data (triangles) show laminar flow on the entire pressure surface, and on the

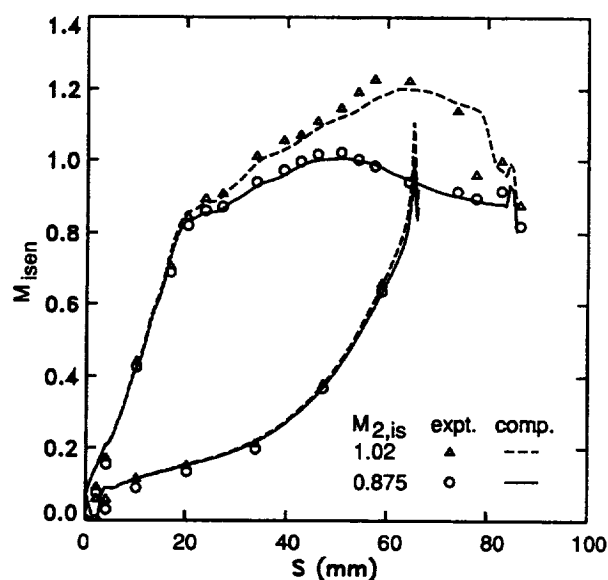


Fig. 3 Computed and measured distributions of isentropic Mach number for the von Kármán Institute turbine vane.

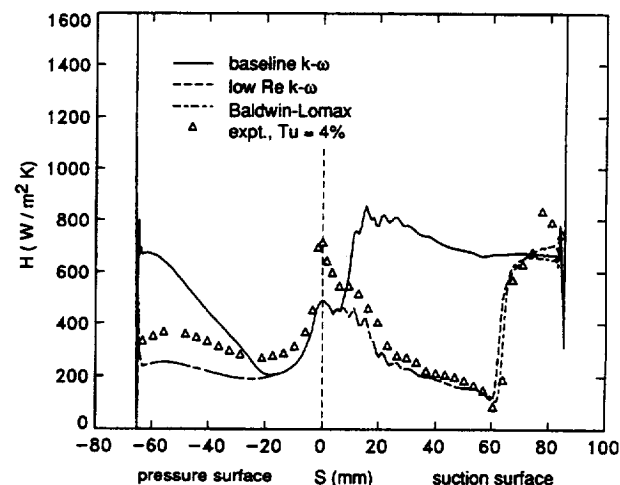


Fig. 4 Surface heat transfer coefficient predicted by three turbulence models.

suction surface up to transition at  $S = 60$  mm. The laminar regions have augmented heat transfer caused by the high free-stream turbulence. This augmentation is not modeled by any of the turbulence models.

The baseline  $k-\omega$  predicted fully turbulent flow on the suction surface and showed transition near  $S = -20$  on the pressure surface, giving high values of  $H$ . The B-L solution agreed closely with the data using the simple transition model proposed in Ref. 1. The low Reynolds number  $k-\omega$  solution also agreed closely with the data; however, the transition point was set by the choice of the inlet values of  $\omega$ , as characterized by the inlet turbulent viscosity. The value of  $\mu_{T,in}$  was varied by about three orders of magnitude to produce solutions that ranged from fully laminar to almost fully turbulent. Only a small range of values of  $\mu_{T,is}$  gave transition near the measured location. The results shown here used  $[\mu_T/\mu_{Lam}]_{in} = 7 \times 10^{-4}$ . It is emphasized that although the location of transition depended strongly on  $\mu_{T,in}$  or  $\omega_{in}$ , the heat transfer in the fully turbulent region was nearly independent of  $\omega_{in}$ .

### Concluding Remarks

Wilcox's  $k-\omega$  turbulence model has been added to a quasi-three-dimensional Navier-Stokes code for turbomachinery.

The model included the effects of rotation, radius change, and stream-surface convergence. An upwind-implicit approximate-factorization scheme was used to solve the turbulence model equations uncoupled from the flow equations. The numerical scheme was robust, but about 18% slower than the B-L model.

Calculations were made of a transonic compressor rotor with significant quasi-three-dimensional effects. The results showed very close agreement between the B-L and  $k-\omega$  models, but both models failed to capture the measured wake spreading. Calculations were also made of a transonic turbine vane with transition and heat transfer. Both turbulence models showed very good agreement with measured surface pressures. Surface heat transfer was predicted reasonably well by the B-L model, considering the simple transition model used. The low Reynolds number  $k-\omega$  model gave similar results, but the predicted transition location was sensitive to inlet values of  $\omega$ . Overall the  $k-\omega$  model behaved well numerically, but predictions were not decisively better than those made with the B-L model.

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